

Study on the phase shift characteristic of the pneumatic Stirling cryocooler

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ABSTRACT

Due to entire pneumatic connection between free piston and free displacer, the motion parameters of them including amplitude and phase shift can actually impact the cooling capacity and overall performance of cryocooler obviously. In this study, the procedure of design and manufacture pneumatic free piston and free displacer (FPFD) Stirling cryocooler had firstly been described in details. Then in order to accomplish study, the experimental bench has been set up based on 80 K@1 W Stirling cryocooler. The effect of the thermodynamic and pneumatic parameters including charging pressure, natural frequency of displacer, damping coefficient of displacer, working frequency on the pressure, displacement and displacer phase shift has been investigated, respectively by means of experimental and theoretical method. In particular, the variation of damping is realized by adjusting the width of clearance cut on the additional damping component, which is screwed on the displacer rod. Similarly, natural frequency of displacer is changed by the extra mass connected on the displacer. Due to the results of experimental study, the optimum working conditions of this Stirling cryocooler for 80 K cold tip temperature are as follows: charge pressure 15 bar, natural frequency of displacer 46 Hz, width of clearance 300 μm and working frequency 43 Hz. In agreement with the optimum working conditions, neighborhood interval of 90° is the ideal working domain for displacement phase shift. Meanwhile, the displacer phase shift should approach to 0° as near as possible and pressure phase shift should also be as small as possible, which have linear relation with non-dimensional damping characteristic of compressor. In view of theoretical study, the expressions of three phase shifts deduced from thermodynamic equation of piston and displacer respectively are expressed as the functions of working parameters, which are verified by the experimental data and consequently can be used as the powerful guidance to optimum seeking.

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1. Introduction

The growing demand for long-wavelength infrared imaging instruments for space observational applications, high temperature superconducting filters and military detector has led to the ongoing development of several second generation Stirling cycle cryocoolers that are scaled up in size from the original Oxford design. Recently, fast cool down, small size, lightweight, low power consumption and high reliability split-type Stirling cryocoolers driven by linear compressor have been used to meet the long operating lift requirements. The free piston free displacer (FPFD) Stirling cryocooler developed initially by the Philips laboratories consists of two-compressor piston driven by linear motors, which make pressure waves and a pneumatically driven displacer piston with a regenerator. In general, the efficiency of the Stirling cryocooler is mainly affected by the efficiency of the linear motor, the resonant frequency of the compressor and expander, the displacement

of the piston and the displacer, and the phase shift between the piston and the displacer.

As shown in Fig. 1, in Stirling cryocooler, the displacer in the expansion cylinder is free and oscillates simply as a result of the gas force acting upon it. This gas force is the direct result of motion of compressor piston driven by the electric force. Due to the complex characteristic of gas spring in the warm and compression space, the phase shift between the displacement of piston and discharge pressure can reveal the performance of linear compressor to some extent. Moreover, the gas force provided by the actuating piston oscillating in the compression cylinder with a phase shift relative to the displacement of displacer indicates the volume change in the expansion space, and its pressure–volume diagram illustrates the work done by the gas on the displacer. This positive work result in the refrigeration effect and the cold finger becomes cold. Finally, the displacement phase shift between piston and displacer reflects the relative position of two moving components literally, but it actually has great effect on the cooling capacity and electric power consumption of cryocooler.

These three phase shift characteristics indicate performance of linear compressor and cryocooler from different viewpoints. Park

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Nomenclature

A	area (m^2)	φ	phase shift between expansion pressure and displacer ($^\circ$)
B	magnetic density (T)	γ	phase shift between the force and electric current ($^\circ$)
C	damping coefficient (N s/m)	Subscripts	
F	amplitude of force (N)	b	bounce space
f	harmonic electric force (N)	c	compressor space
i	current (A)	d	displacer
k	stiffness (N/s)	e	expansion space
M, m	mass (kg)	ee	electric
p	pressure (Pa)	g	gas spring
Q	refrigeration capacity (W)	h	hot space
t	time (s)	m	mechanical stiffness or damping
COP	coefficient of performance	mean	mean value
x	displacement (m)	ideal	ideal value
X	amplitude of displacement (m)	n	natural frequency
V	buffer volume (m^3)	o	mean value or operating frequency
Greek letters		p	compressor piston
α	polytropic coefficient, $1(\text{isothermal}) \leq \alpha \leq \gamma(\text{adiabatic})$	r	rod
ω	angular frequency (rad/s)		
ψ	displacement phase angle ($^\circ$)		

[1] had measured the cool-down characteristics of the cold end with laser displacement sensor in the expander of the Stirling cryocooler. The amplitude of displacer's pressure and displacement had been presented with the variation of charging pressure (15–25 bar) and operating frequency (40–55 Hz). when the displacement phases shift is 45° , optimum operating frequency was finally found, which was decided by many factors including resonant frequency of expander and friction and flow resistance. Kornhauser [2] had developed an improved linear dynamic model (anelastic model) of gas springs in the Stirling engine, which dealt the traditional isothermal model in parallel with adiabatic model and could predict the phase shift relative to discharge pressure of compressor and piston displacement, stroke amplitude and damping characteristic theoretically. Gaunekar [3] had set up a mathematical model aiming to analyze the performance characteristic of cryocooler systematically in theory, although many assumptions had been adopted, some results can still be used as the guide of design.

In this study, pressure, displacement and displacer phase shift characteristics have been investigated respectively by means of experiment under different conditions, such as working frequency, charged pressure, natural frequency and motion damping coefficient of displacer. In addition, the corresponding theories of them

have been deduced from dynamic equations of piston and displacer, which reveal the relationship between the working parameters and three phase shifts respectively. The experimental results of the performance and optimum operating conditions of the Stirling cryocooler according to the variations of the above phase shifts are presented as well. This phase shift is the powerful tool to the performance evaluation and optimum design of Stirling cryocooler.

$D_1 \sim P_1$ Phase shift between piston displacement and discharge pressure (pressure phase shift ψ_{pp})

$D_1 \sim D_2$ Displacement phase shift between piston and displacer (displacement phase shift ψ_{pd})

$P_1 \sim D_2$ Phase shift between discharge pressure and displacer displacement (displacer phase shift ψ_{dd})

2. Design and manufacturing of the Stirling cryocooler

2.1. Design of Stirling cryocooler

In terms of the physical process, the pneumatic Stirling cryocooler can be regarded as the two-freedom damping vibration system as a whole. The working gas in compression, expansion chamber and regenerator accomplishes the thermodynamic process that is

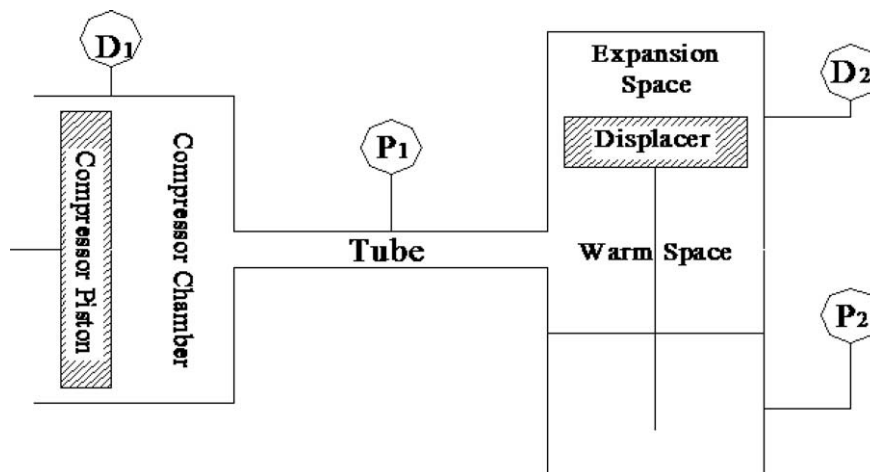


Fig. 1. Schematic diagram of PFED Stirling cryocooler.

actually coupling tightly with the dynamic working conditions. The equation describing the motion of piston and displacer [4] is as follows

$$M_d \ddot{x}_d = (A_e - A_r)p_h - A_e p_e + A_r p_b - k_{md} \dot{x}_d - C_{md} \ddot{x}_d \quad (1)$$

$$M_p \ddot{x}_p = (p_b - p_c)A_p - k_{mp} \dot{x}_p - C_{mp} \ddot{x}_p + B_{il} \quad (2)$$

In above equations, the gas pressure, damping factor is essentially nonlinear term which needs very complex iteration process before getting the convergent solution. Getting precise mathematic solution of whole Stirling cryocooler control equations is a very heavy burden to an engineer. In practical design, the isothermal model [5] always adopted to simplify the thermodynamic model and subsequently explicit expression of thermodynamic parameters can be obtained. Moreover, the motion of piston and displacer coupled tightly with each other by the pneumatic pressure term is always uncoupled by means of sine wave assumption of these two displacements. Meanwhile the first three terms in the right side of Eq. (1) and the first term in the right side of Eq. (2) reflecting the pneumatic force of expansion and compression space can be equivalently simplified to gas spring and equivalent hysteresis loss. The gas spring constant is given by [6]

$$k_g = \frac{\alpha P_{mean} A^2 x}{A(x_m - x) + V} \quad (3)$$

Then resonant frequency of piston and displacer is as follows:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (4)$$

For the compressor, the gas stiffness is bigger than that of mechanical spring. So Eq. (3) should be involved in the natural frequency calculation according to Eq. (4). In contrast, the gas spring stiffness of displacer is weaker than that of spring, so it can be ignored.

The ideal refrigeration (PV power) of cryocooler can be written as the following equation [1]:

$$Q_{ideal} = \pi f P_d A_d X_d \sin \varphi \quad (5)$$

As shown in Eq. (5) the ideal refrigeration is greatly affected by the operating frequency, the amplitude of pressure, the amplitude of

the displacer and the phase shift between the pressure and displacement.

2.2. Manufacture of Stirling cryocooler

The FPDF Stirling cryocooler for case study consisted of three major parts (Fig. 2): the linear compressor module, the expander module and electric power supply and control module. The linear compressor consisted of aluminum alloy base, inner and outer iron yokes, permanent magnets (NdFe35), coils (copper wire), cylinders, pistons and flexure springs. The compressor was based on the well-proven 'Oxford' principles of spiral flexure springs and non-contacting clearance seals. The machine was a compact moving piston design, with the piston and cylinder located within the core of the magnetic circuit of a moving coil motor. The springs were the only component subjected to significant fatigue loading, and there were routinely batch and sample tested at their full stroke in excess of 4×10^9 cycles. In addition, strain-stress test conducted indicated the linear stiffness relationship. The structure of the compressor consists of two opposite parts that were inherently well balanced with the two halves mounted in line and operated in anti-phase. As a result, very low levels of self-induced vibration had been obtained. In terms of material selection, non-metallic materials used in the assembly such as plastics, adhesives and gasket were selected from an existing knowledge base of materials with a low out-gassing rate. As for metal material, the strength, stiffness, thermal expansion and processing characters had been considered and finite element analysis of some important components had even been done. Meanwhile, geometric tolerances and surface requirement should commensurate with the functional demands [7].

The expander module consisted of a displacer, a regenerator in the displacer (phosphor, tin bronze or stainless steel wire screen), a displacer cylinder, a spring and a heat exchanger. The displacer with the regenerator was actuated by the pressure difference between the expansion space and the warm space. In order to reduce the mass of displacer, the non-metal material had been adopted to process the shell of regenerator. Meanwhile, the axial heat conduction had almost been avoided due to very low coefficient of heat conductivity. Moreover, in order to reduce the axial conductivity

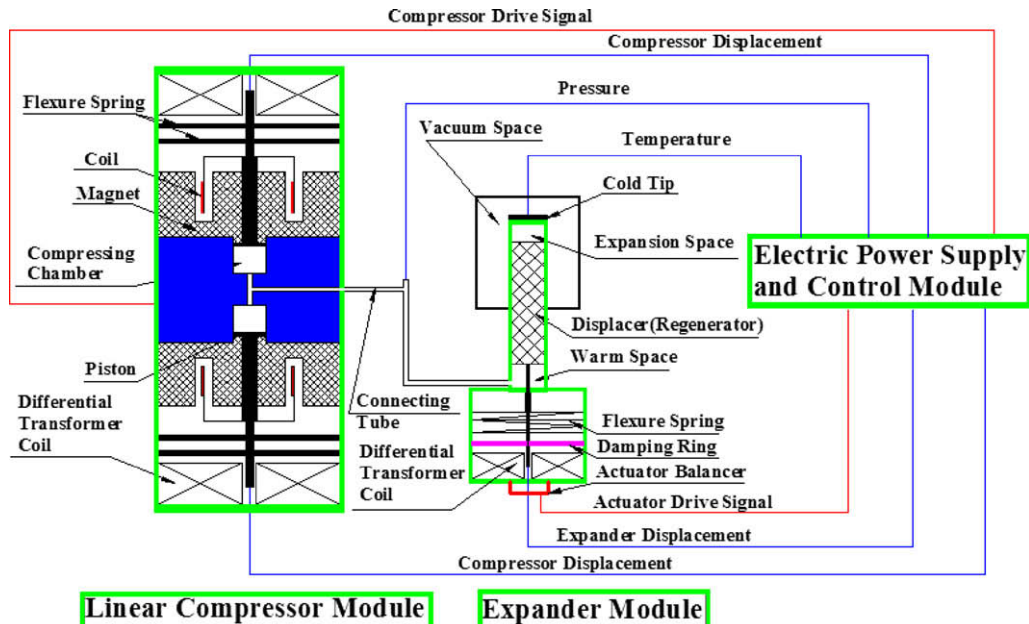


Fig. 2. Schematic diagram of Stirling cryocooler.

loss of the displacer cylinder, the high strength metal thin wall tube with very small section area had also been used. In terms of the regenerator, material property, mesh, porosity, weave style were the main factors should be considered. Pressure drop and heat transfer characteristic of regenerator related closely with above parameters could affect the performance of cryocooler obviously. Determination of these parameters highly depended on the accumulated data and empirical correlation expressions [8].

One of key components to the overall efficiency of a cryocooler was how well heat was transferred from the heat load to the helium in the cold end of cooler. The more efficiently this heat transfer occurred, the lower the temperature difference between the heat load and helium in the cold end would be. This resulted in the helium not having to be as cold as more efficient cooler. There were essentially three resistances to this flow of heat: the first one was the interface resistance between the heat load and the heat acceptor portion of the cold end, the second one was the conduction resistance across the heat acceptor, and the last one was the convection resistance from the heat acceptor to the helium gas inside the cold end. In cryocooler, the first resistance was mainly composed of the contact thermal resistances between the heat load and cold end, which could be reduced by coating the heat transfer enhance grease or cryogenics glue. The red copper cold end with very high coefficient of thermal conductivity was welded directly with the displacer cylinder. It was the last resistance that is the dominant term in the overall resistance, this resistance was defined as $1/(hA)$, where h = convective heat transfer coefficient, and A = heat transfer area. A special small-hole structure was adopted to increase the velocity of helium scouring the inner face of cold end. So the convective heat transfer was enhanced.

The reliable, lightweight, compact, high efficiency electronic controller installed in a separate box contains a moving mass position signal processor for the compressor and displacer respectively, power supplies, telemetry interface, vibration cancellation circuitry, and logic circuit. There are two control models could be chose. The first one was the close loop control which could be used in practical engineering. After inputted the digital single from A/D converters of the two LVDT sensors, the servo control system, in which a single-chip microcontroller (8031) had been used as the main controller, determine the operating parameters of the moving masses including direct current bias, frequency, phase between compressor and displacer. These parameters then were processed by program stored in the chip, which followed the structured programming method and includes start-up block, main control block, pressure force process block and self-test block telemetry block. After iteration calculation, the feed back signal was generated and subsequently input parameters could be regulated. The other was open loop control model, which was more suitable to the experimental study. Many above feedback processes and self-regulating functions had been cancelled. Various parameters could be adjusted manually [9].

3. Experimental facility and procedure

3.1. Experimental facility

The tested machine is a one-stage Oxford split-Stirling cryocooler constructed at Cryogenic Laboratory in Shanghai Institute of Technical Physics (SITP), whose geometrical dimensions and operation conditions are list in Table 1. Fig. 3 shows a schematic diagram of the experimental apparatus of the FPDF Stirling cryocooler. Two piezoelectric dynamic pressure sensors were used to monitor the pressure oscillations at the outlet of the compressor and buffers of displacer. Meanwhile, two linear variable-differential position transformers (LVDT) were mounted at the end of the compressor and displacer respectively for the displacement measurement of the pistons, enabling the volume change to be determined subsequently according to the driven mechanism. A platinum resistance thermometer being calibrated was attached to the cold head to measure the temperature of the cold end. Film heater attached on the cold end has counteracted the refrigeration capacity of cryocooler that is equal to the power of heater. The apparatus of the Stirling cryocooler, except for the components of the room temperature region, was connected to the vacuum flange during the experiment; a vacuum chamber was connected to a high vacuum pump under a pressure of 10^{-5} Pa. The high vacuum pump system consists of a rotary roughing pump, a turbo-molecular pump and vacuum gauges.

The following operation parameters can be adjusted, including voltage amplitude, voltage frequency, charged pressure, temperature of cold tip and refrigeration capacity. Moreover, the geometrical parameters including the mesh of regenerator, length of connecting tube and the stiffness of spring can also be adjusted according to the experiment demands. In this experiment study, in order to investigate the characteristic of phase shift further, besides the above common measuring and regulating measures, the following special methods have been adopted.

- (1) An extra pneumatic damping ring has been designed and installed on the expander assembly. Different damping coefficient can be obtained by adjusting the width of clearance cut on the ring.
- (2) In order to get the different natural frequency of expander assembly, Additional mass has been screwed on to the expander assembly.

The following tests were undertaken for the experimental performance analysis and comparative study of cryocooler.

- (1) The amplitudes of the displacer's stroke and phase shift under different temperatures.
- (2) The cooling capacity characteristics under the natural frequency of the displacer.

Table 1
Dimension and operating conditions of FPDF Stirling cryocooler.

Working fluid	Helium		
Charge pressure (p)	0.8–1.5 MPa		
Working conditions	Ambient temperature 298 K, relative humidity 60%, cold tip temperature 80 K~200 K		
Parameters of compressor		Parameters of displacer	
Dimensions of compressor	ϕ 120 mm $\times$ 170 mm	Dimensions of displacer	ϕ 160 mm $\times$ 148 mm
Stiffness of piston spring (k_p)	1.8 N/mm	Stiffness of displacer spring (K_d)	3 N/mm
Mass of piston (m_p)	120 g	Mass of displacer (m_d)	22 g
Input voltage	28 V	External Diameter of connecting tube	2.4 mm
Input excitation frequency (f)	35–44 Hz	Length of connecting tube	220 mm
Specific thrust	14 N/A		

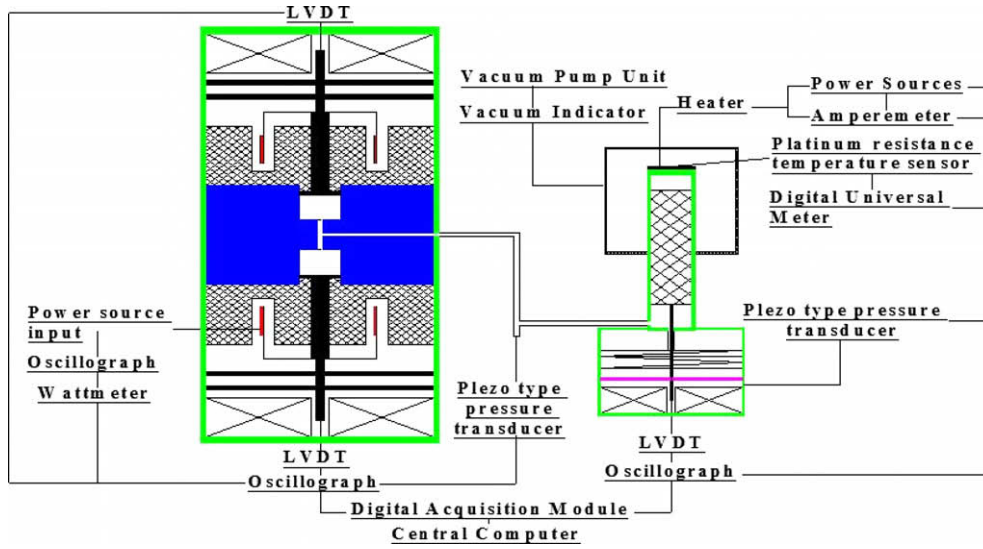


Fig. 3. Experimental apparatus of the FPF Stirling cryocooler.

- (3) The displacer's amplitudes under various charging pressures, the amplitudes of pressure and of operating frequency.
- (4) The phase shifts between the displacements of the piston and the displacer under different operating frequencies.

3.2. Experimental procedure

In the process of experiment, change of charging pressure and additional mass of displacer involve assembly and disassembly of cryocooler. In reality, mechanical damping is very sensitive to the assembly technology. So in order to guarantee recurrence of experiment, the damping characteristic of piston and displacer must be measured and subsequently adjusted under the unload conditions. Indispensable steps should be implemented in following sequence, such as assemble the Stirling cryocooler, damping test, leak detection, evacuating cryocooler, gas charging, evacuating vacuum chamber, connecting the cryocooler with the measure instrument and control box, regulating the operating parameters, collecting the performance parameters, finishing the experiment, closing the electrical power, rewarming cryocooler, change components.

4. Theory of phase shifts

4.1. Displacement phase shift between displacer and piston

Motions of displacer and piston can approximately be considered as the harmonic wave, and are given, respectively, by

$$x_d = X_d \cos(\omega t + \psi_{pd}) \quad (6)$$

$$x_p = X_p \cos(\omega t) \quad (7)$$

According to the theory of free piston Stirling refrigerator developed by Jonge [10], the pressure fluctuation in the variable compression and expansion volumes are expressed as

$$p_c = p_m + K_p x_p + K_d x_d + C_{pc} \dot{x}_p + C_{dc} \dot{x}_d \quad (8)$$

$$p_e = p_m + K_p x_p + K_d x_d + C_{pe} \dot{x}_p + C_{de} \dot{x}_d \quad (9)$$

This assumption is based on the ignorance of the effect of leakage, temperature variations on the pressure. Also the flow is regarded as the laminar flow so as to be liberalized conveniently.

Moreover, this simple description of the pressures is only valid if all constants K_p , K_d , C_{pc} , C_{dc} , C_{pe} and C_{de} are independent of x_d and x_p , respectively. Actually, from our previous experimental data and published information [11], the stiffness of gas spring (K_p) in compression space is so big that has same magnitude with stiffness provided by mechanical spring. In addition, this stiffness is very sensitive to the stroke of piston. In order to reflect this characteristic in Eq. (8), the K_p should be divided into two parts in reference to Eq. (3), the first one ($\bar{K}_p = P_{mean} A_p^2 / V_0 B$) has no relationship with the piston stroke; another one ($\bar{K}_p = P_m A^2 \left[\frac{\alpha}{A_p (x_m - x_p) + V} - \frac{1}{V_0 B} \right]$) is the function of piston stroke. In contrast, the gas spring effect is far weaker in displacer than that in piston, so K_p in Eq. (9) equal approximately to $\bar{K}_p$. So subtracting P_c from P_e , the pressure difference is

$$\begin{aligned} p_e - p_c &= (C_{pe} - C_{pc}) \dot{x}_p + (C_{de} - C_{dc}) \dot{x}_d - \bar{K}_p (X_p) x_p \\ &= C_{pp} \dot{x}_p + C_{dd} \dot{x}_d - \bar{K}_p (X_p) x_p \end{aligned} \quad (10)$$

The ideal refrigeration effect and ideal power input, the expressions for which are given by Jonge [3] can be rewritten as

$$Q_{eo} = n \oint p_d dv_d = -\pi f K_p X_p X_d A_d \sin \psi_{pd} \quad (11)$$

$$P_{ci} = n \oint p_p dv_p = \pi f K_d X_p X_d A_p \sin \psi_{pd} \quad (12)$$

The ideal coefficient of performance (COP_{ideal}) may now be written as

$$COP_{ideal} = Q_{eo} / P_{ci} = K_p A_d / K_d A_p \quad (13)$$

It should be noted that, though the refrigeration effect is directly proportional to $\sin \psi_{pd}$, the ideal COP_{ideal} is independent of ψ_{pd} .

The area of displacer rod is comparatively smaller than that of displacer, so if the area of displacer rod (A_r) is ignored, the Eq. (1) can be transform into

$$M_d \ddot{x}_d = A_e (p_h - p_e) - k_{md} x_d - C_{md} \dot{x}_d \quad (14)$$

in which p_h can be expressed as $p_h = p_c - \Delta p_{hc}$. As far as the construction is concerned, the pressure difference (Δp_{hc}) between hot and compression chamber can be attributed to two factors, the first one is the friction loss in the connecting tube, the second one is the compressibility and flow loss effect of gas in dead volume if it has. According to the published investigation [11] the pressure ampli-

tude declines about 5% across the connecting tube, which can almost be ignored in this calculation. So the Eq. (14) can be transform into

$$M_d \ddot{x}_d = A_e(p_c - p_e) - k_{md}x_d - C_{md}\dot{x}_d \quad (15)$$

Substitute Eq. (10) into Eq. (15), the expression is

$$M_d \ddot{x}_d + C_d A_e \dot{x}_d + k_{md}x_d = -A_e C_{pp} \dot{x}_p + A_e \ddot{x}_p (X_p) x_p \quad (16)$$

in which $C_d = C_{md}/A_e + C_{dd}$. Avail of the harmonic wave assumption (Eqs. (6) and (7)), the displacement phase shift and amplitude ratio can be deduced from the Eq. (16) which is as follow:

$$\psi_{pd} = \arctan \left(\frac{(\omega_d^2 - \omega^2)M_d}{\omega C_d A_e} \right) + \arctan \left(\frac{\ddot{x}_p}{\omega C_{pp}} \right) \quad (17)$$

$$\frac{X_p}{X_d} = \left[\frac{M_d^2(\omega_d^2 - \omega^2)^2 + (\omega C_d A_e)^2}{(\omega C_{pp} A_e)^2 + (\ddot{x}_p A_e)^2} \right]^{1/2} \quad (18)$$

From previous viewpoint [10], only under the conditions of $\omega < \omega_d$ and $\psi_{pd} > 0$ can the machine run as a refrigerator. But according to experimental phenomenon, the output refrigeration capacity is actually not decline to zero, which contradicted with above theoretical prediction. The second term of right side of Eq. (17) explains this controversy, which leads to a positive angle and has a positive relationship with the stiffness of gas.

4.2. Phase shift between the system pressure wave and displacement of piston

With respect to the structure of Stirling cryocooler, the pressure fluctuation attributed to four predominant causes [10], there are piston position determining the total volume, displacer position determining the amount of gas in the expansion and compression spaces, piston and displacer movement giving rise to gas flow, causing pressure differences in the space. Similarly, the pressure wave shift can also attribute to four principle reasons, such as property change with the temperature, energy hysteresis and damping effect as well as energy store and conversion effect.

In gas spring model [2], gas spring force and displacement are related to pressure and volume. the differential equation that is transformed from Eq. (2) describing the performance of the modelled gas spring more clearly is

$$F_p = k_{mp}x_p + C_{mp}\dot{x}_p + M_p\ddot{x}_p - f_e \quad (19)$$

In which $f_e = F_e \cos(\omega t + \gamma)$. Substituting the sinusoidal displacement of piston into above equation, the gas spring phase, magnitude and dissipation are thus found to be

$$\psi_{pp} = \arctan \left[\frac{\omega C_{mp}X_p - F_e \sin \gamma}{(k_{mp} - \omega^2 M_p)X_p - F_e \cos \gamma} \right] \quad (20)$$

$$\hat{F} = \frac{|F|}{X_p k_{mp}} = \sqrt{\left(1 - \frac{M_p \omega^2}{k_{mp}} - \frac{F_e \cos \gamma}{k_{mp} X_p}\right)^2 + \left(\frac{\omega C_{mp}}{k_{mp}} - \frac{F_e \sin \gamma}{k_{mp} X_p}\right)^2} \quad (21)$$

$$\hat{D} = \frac{f P_c d V_c}{P_m V_m \left(\frac{V_c}{V_m}\right)^2} = \frac{\pi(\omega C_{mp}X_p - F_e \sin \gamma)}{(k_{mp} - \omega^2 M_p)X_p - F_e \cos \gamma} \quad (22)$$

From above equations, the relationship of ψ_{pp} and $\hat{D}$ is $\hat{D} = \pi \tan \psi_{pp}$ the relationship of ψ_{pp} and $\hat{F}$ is $\hat{F} = \sec \psi_{pp}$. These expressions reveal that the dimensionless amplitude and dissipation is the monodromy function of phase shift. With the increase of the ψ_{pp} , the non-dimension damping ($\hat{D}$) will increase correspondingly, meanwhile the non-dimension amplitude ($\hat{F}$) will decrease.

4.3. Relationships between the displacer displacement and system pressure

Eq. (14) can also be used to investigate the phase shift between the system pressure and displacer displacement.

$$M_d \ddot{x}_d + k_{md}x_d + C_{md}\dot{x}_d = A_e(p_h - p_e) \quad (23)$$

From Eq. (10), pressure difference ($p_h - p_e$) is actually the harmonic function of displacement of piston and displacer as well as their first order differential. Applying the Fourier expansion, this pressure difference can approximately expressed as

$$p_h - p_e = P_{he} \sin \omega t \quad (24)$$

Substituted the Eq. (24), the phase shift is

$$\psi_{dd} = \arctan \frac{k_{md} - \omega^2 M_d}{\omega C_{md}} = \arctan \frac{M_d(\omega_d^2 - \omega^2)}{\omega C_{md}} \quad (25)$$

5. Experimental results and discussion

Fig. 4 shows the real displacement wave of piston and displacer. According the characteristic of moving coil linear motor, the exciting force (Bil) has positive relationship with the input voltage. So by adjusting the output voltage of regulated power source, the amplitude of compressing piston can be controlled. With respect to the frequency of displacement, because the Stirling cryocooler can actually be regarded as the single input and two output vibration system, the frequency of response (x_d, x_p) equals to that of

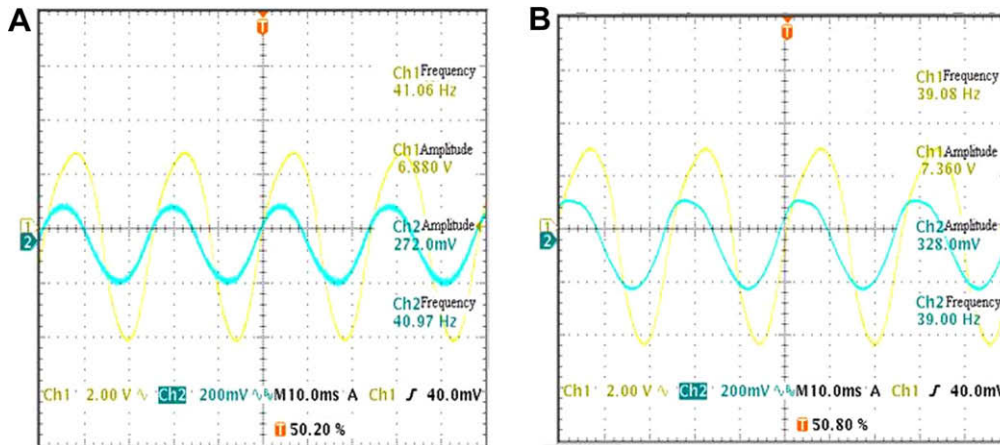


Fig. 4. Real time displacement wave of the piston and displacer, when (A) Operating frequency is 41.06 Hz, width of damping clearance is 300um, 10 bar (B) Operating frequency is 39.08 Hz, width of damping clearance is 300 um, 14 bar.

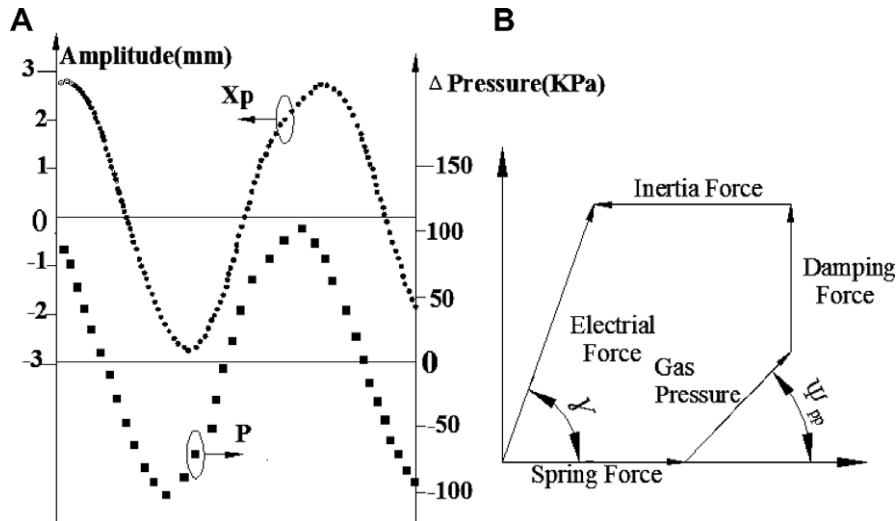


Fig. 5. (A) Real wave of displacement of piston and gas pressure, when the operating frequency is 37 Hz, width of clearance is 450 μm . (B) Force vector diagram of piston.

exciting force (Bil). However due to the strong nonlinear pneumatic force driving the movement of displacer, the distortion caused by high frequency harmonic wave has happened clearly. The phase shift of these two displacement influenced by the geometrical and operating parameters can be measured easily from above figures.

Fig. 5A shows the displacement wave of piston lagging behind the gas pressure wave. Similar to the displacement wave of piston and displacer, the harmonic deformation of pressure exists. Deduced from Eq. (2), the vector diagram of force applied on the piston (shown in Fig. 5B) reveals the relationship of magnitude and phase angle of them. It is clear that the inertia force parallel with the spring force that is vertical to the damping force. The phase shift (ψ_{pp}) in Fig. 5B can, respectively influence the spatial position and magnitude of force vectors. From this oscillogram, when the operating frequency is 37 Hz and width of clearance is 450 μm , the phase shift is 46.6°.

5.1. Effect of cold tip temperature on the phase shift

In the cooling process, with the decrease of the cold tip temperature, average temperature of helium in the regenerator decline gradually, which result in the declination of pneumatic driving force determined by the pressure drop across the regenerator. As a consequence, the stroke of displacer decreases consequently. For instance, under the ambient temperature (293 K) dynamic viscosity of helium is $19.52 \times 10^{-6} \text{ N s/m}^2$ and the density is 2.377 kg/m^3 . In contrast, when the temperature is 80 K, dynamic viscosity of helium become $15.2 \times 10^{-6} \text{ N s/m}^2$, which decrease about 22% and density become 8.936 kg/m^3 , which is 3.759 times of that of ambient temperature. When the temperature of cold tip decreasing from 230 to 63 K and keeping constant piston stroke (3.8 mm), as a result of variation of property displacer stroke decrease correspondingly from 1.45 to 0.95 mm. So in order to shorten the period of cooling process in practical operation, the piston stroke should be increased by request of enough displacer strokes.

In terms of displacement phase shift between the piston and displacer, due to spring stiffness sensitive to piston stroke, natural frequency coupling with damping coefficient (C_{pp}) [11] increasing simultaneously with the decrease of cold tip temperature, in other words, numerator and denominator of the second term in Eq (17) increasing at the same time, the phase shift just increase in a little degree consequently. For example, for working condition (A), when temperature decrease from 270 to 60 K, the phase shift vary from

87.5° to 89.5°, just increase 2.28%. From Eqs. (11) and (12), if ignoring the effect of varying phase shift, in cooling process the refrigerating capacity and efficiency has positive relationship with the magnitude of stroke approximately. So for Strling cryocooler, if the low temperature were needed, the enough strokes of piston and displacer should take measure to be provided and the collision between piston and cylinder should be avoided simultaneously.

The resonance characteristics [12] of the compressor also change somewhat between ambient and cryogenic temperatures, which meanly attributes to the stiffness of the gas spring drops with decreasing gas pressure as gas density increases in the cold finger. Note that the damping contrastingly increases somewhat at cryogenic temperatures. Meanwhile the damping also increases somewhat with increasing stroke. So as shown in Fig. 6, the phase shift increase with the decrease of temperature following the principle given by Eq. (20). Furthermore, the non-dimension damping coefficient given by Eq. (22) increases about 10%. So it can be concluded that the working performance actually deteriorate under the low temperature working condition.

With reference to the displacer phase, C_{md} will increase in consequence of lowering temperature. Meanwhile the gas spring stiffness of displacer keeps almost constant, so the phase will decrease as shown Fig. 7. The increasing ratio of damping can be

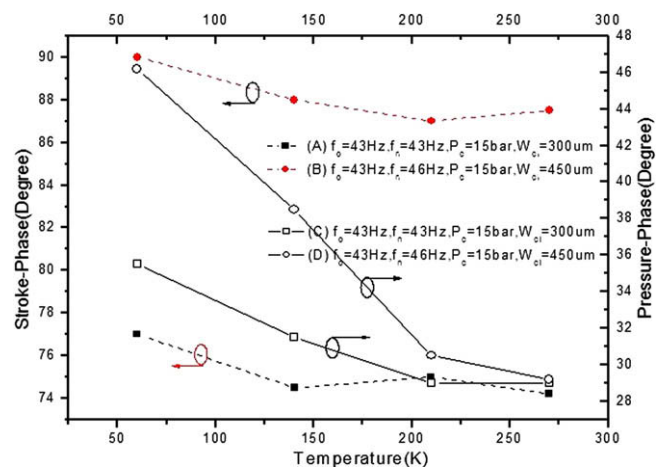


Fig. 6. Displacement phase shift between displacer and piston & phase shift between system pressure and the displacement of piston with the variation of cold tip temperature.

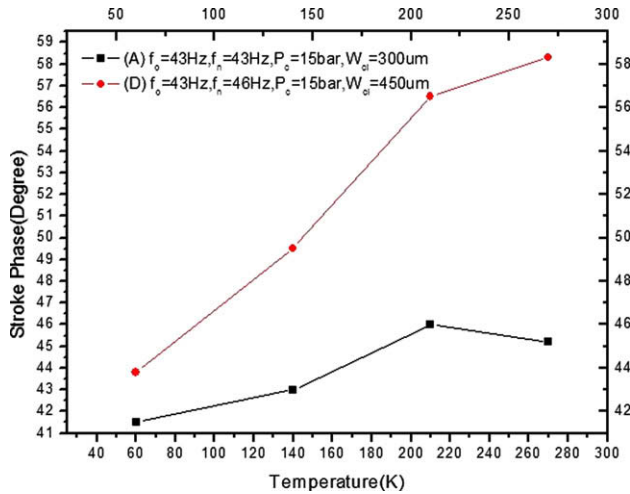


Fig. 7. Phase shift between displacer displacement and the system pressure with the variation of cold tip temperature.

deduced from the variation of phase shift. For instance, for condition (A), when temperature varying from 270 to 60 K, the damping coefficient (C_{md}) increase 1.6885 times.

From another viewpoint, it can be found that as for three phase shifts, that of the displacer with wider clearance (450 μm) is bigger than that with 300 μm clearance. It reveals that the less the system damping has, the bigger phase shift will be.

In general, cold tip temperature causing the variation of helium property increases damping of both piston and displacer. But it seems that the damping of piston is more sensitive to the stroke than property variation. And movement characteristic of piston is controlled in larger degree by the external factors including electric voltage and frequency. In contrast, the motion of displacer depends clearly on damping coefficient instead of stroke.

5.2. Effect of operating frequency on the phase shift

Frequency adjustment can be realized conveniently by the variable frequency power source. In this series experiment, only frequency of input electric power is adjusted. For the displacer with 300 μm clearance, while frequency varying from 43 to 37 Hz, the strokes of piston increases from 2.36 to 3.983 mm correspondingly.

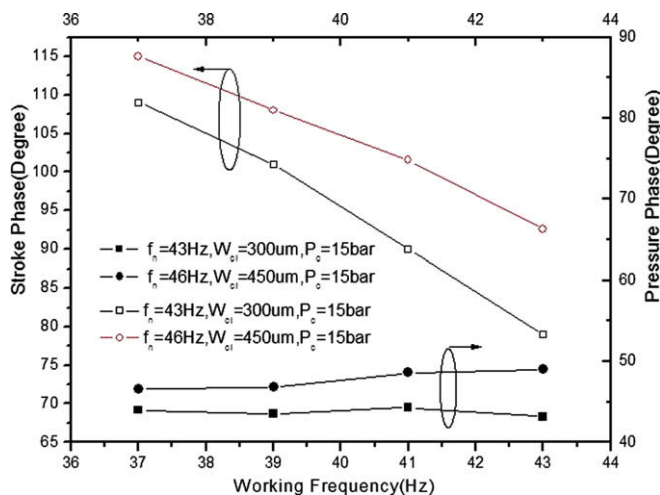


Fig. 8. Displacement phase shift between displacer and piston and phase shift between the system pressure and displacement of piston with the variation of working frequency.

So referring to the displacement phase shift, although the damping (C_d) decrease with the declination of stroke of displacer, working frequency is actually the main influencing factor which can be deduced from Eq. (17). As shown in Fig. 8, for the displacer with 450 μm clearance and 46 Hz natural frequency stroke phase decrease from 115° to 92.6° while working frequency varying from 37 to 43 Hz. When the working frequency is 41 Hz, it also can be drawn from Fig. 8 that the more natural frequency and wider clearance has, the bigger phase shift will be. This tendency obviously coincides with that predicted by Eq. (17).

Conversely, the pressure phase shift increase slightly with increase of working frequency. For instance, the phase shift increases about 5.15% when working frequency increase from 37 to 43 Hz for displacer with 450 μm clearance and 46 Hz natural frequency. For compressor, increasing frequency brings on decreasing stroke (X_a), subsequently the damp (C_{mp}) and stiffness of piston (K_{mp}) will decline. From Eq. (20), the comprehensive effect of above factors leads to increase of phase shift. Meanwhile, the almost fixed the dimen-

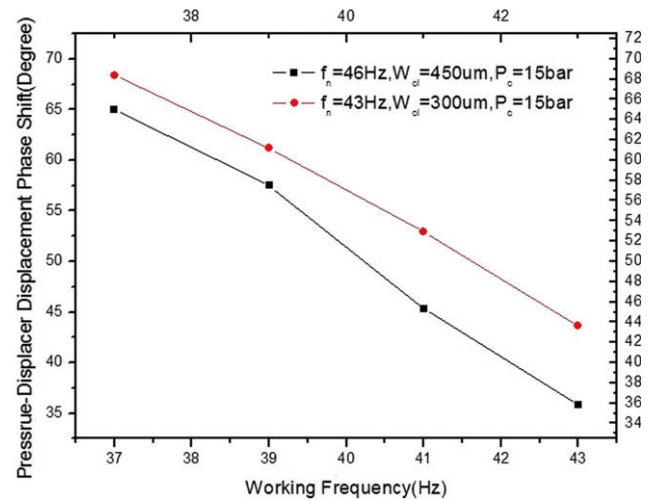


Fig. 9. Phase shift between the displacer displacement and system pressure with the variation of working frequency.

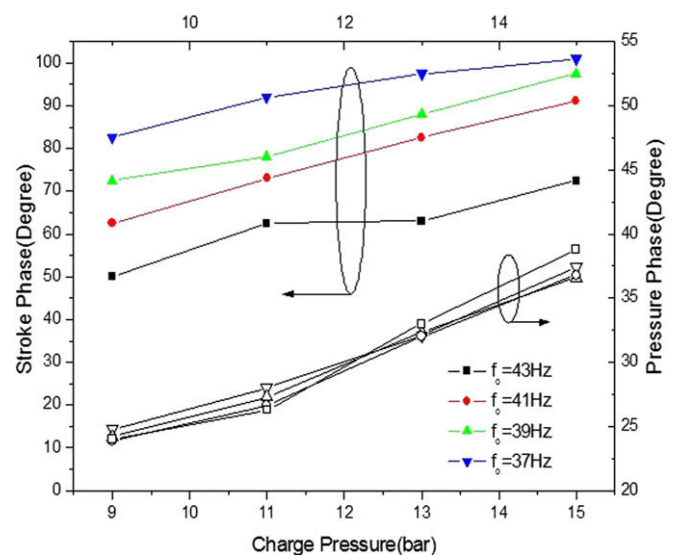


Fig. 10. Displacement phase shift between displacer and piston and phase shift between the system pressure and displacement of piston with the variation of charge pressure, while $f_n = 46 \text{ Hz}$, $W_{ci} = 300 \mu\text{m}$.

sionless damp shown in Eq. (22) accounts for the performance of compressor just decline a little with variation of working frequency.

From classical vibration theory, when working frequency is less than natural frequency, the displacer phase shift will rise with increase of working frequency. Higher natural frequency or lower damping coefficient will lead to bigger phase shift. It also can be seen that the resonant frequency is actually less than the natural frequency and the difference between them depends on damping coefficient. From Fig. 9, it is clearly that less damping and large natural frequency leads to less phase shift, which conforms to the prediction of Eq. (25). However, it exists a paradox seemingly between Fig. 9 and Eq. (25) that when the working frequency is 43 Hz, the phase shifts of displacer whose natural frequency is 43 Hz do not equal to zero (Fig. 9), which can be deduced from Eq. (25). This contradiction actually attribute to the traditional calculation method

used in primary design, that is for displacer the mechanical spring stiffness is just involved in the natural frequency calculation. But in fact, the gas in the hot and expansion chamber can provide stiffness to some extent, which is far less than that of mechanical spring and ignored at most cases. This gas stiffness is also the function of working parameters.

5.3. Effect of the charge pressure on the phase shift

For displacer with $f_n = 46$ Hz, $W_{cl} = 300$ μm , when working frequency is 41 Hz and charge pressure varies from 9 to 14 bar, the stroke of piston decrease correspondingly from 3.93 to 3.6 mm and the stroke of displacer also decrease about 15%. This tendency can also applied to other working frequencies [12,13]. From Eq. (3),

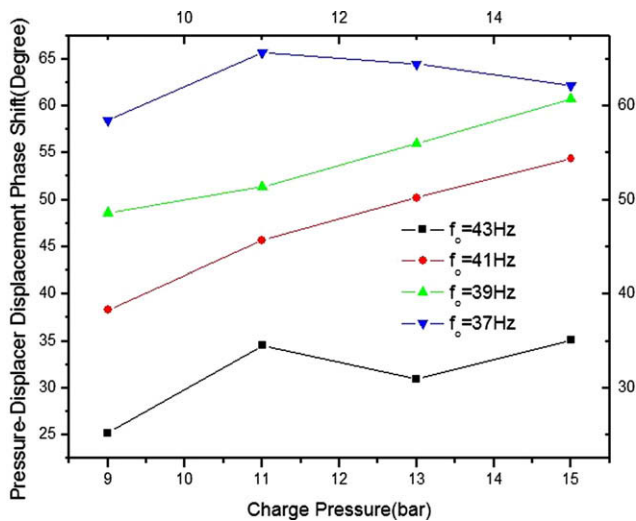


Fig. 11. Phase shift between the displacer displacement and system pressure with the variation of charge pressure, while $f_n = 46$ Hz, $W_{cl} = 300$ μm .

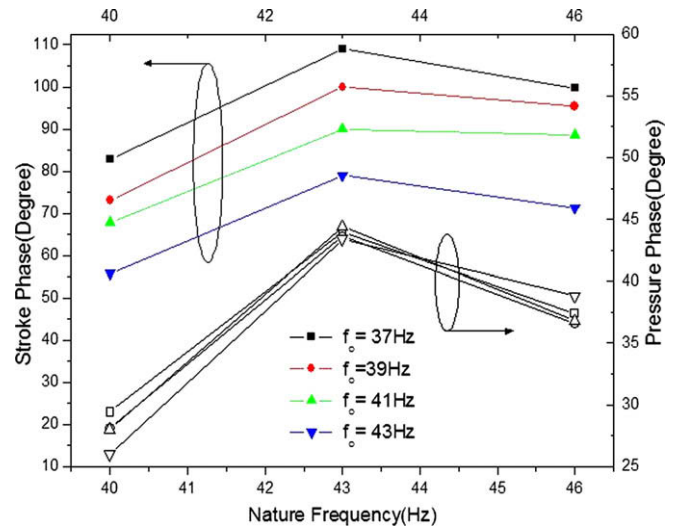


Fig. 13. Displacement phase shift between displacer and piston & phase shift between the system pressure and displacement of piston with the variation of displacer's natural frequency, while $P_c = 15$ bar, $W_{cl} = 300$ μm .

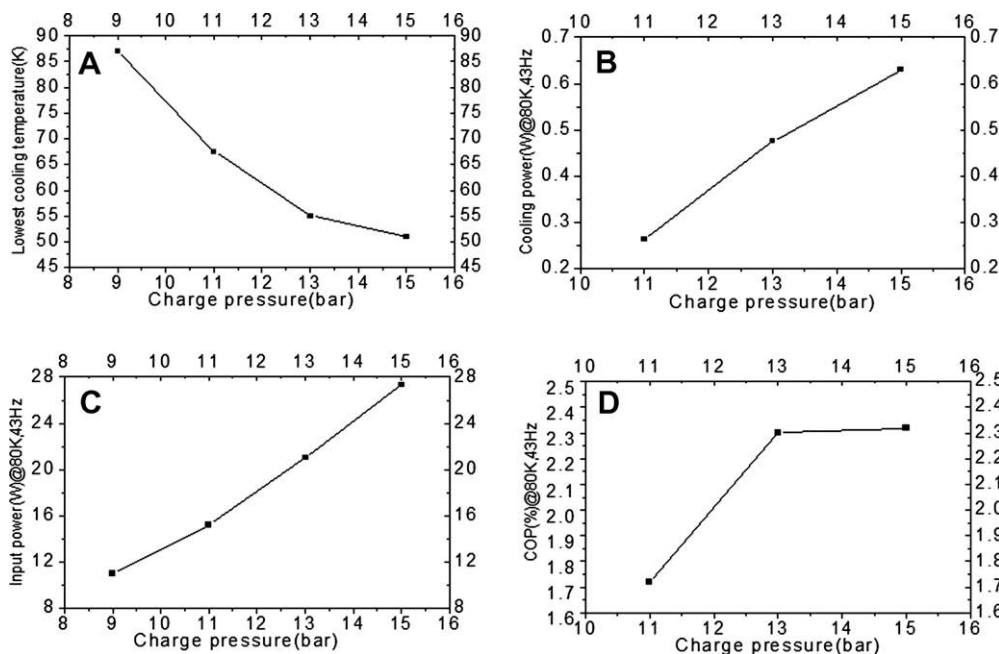


Fig. 12. When $f_n = 46$ Hz, $W_{cl} = 300$ μm , with the variation of charge pressure (A) the lowest reached temperature, (B) cooling capacity when the temperature is 80 K, (C) input power when the temperature is 80 K, working frequency is 43 Hz, (D) COP when the temperature is 80 K, working frequency is 43 Hz.

the stroke and charge pressure has opposite effect on the gas stiffness and damping coefficient. However as shown in Fig. 10, these two phase shift have the positive relationship with the charge pressure. Pressure phase shift shown in Eq. (20) have negative relation with the piston stroke. As for the gas stiffness of piston, the increasing mean pressure and dead volume have conversely effect on the value of stiffness (seen in Eq. (3)). So the gas stiffness of compressor keeps constant as a whole. On the contrary, natural frequency of displacer has not same sensitivity to stroke as that of piston. From Figs. 10 and 11, it can be deduced that increasing phase shift imply that the effect of decreasing stroke of displacer on the damping coefficient is bigger than that of increasing mean pressure. In the end, the displacer damping will decline. For instance, from Fig. 11 when working frequency is 41 Hz, with the variation of charge pressure from 9 bar to 15 bar, the displacer phase shift increase from 38.3° to 54.4° . The corresponding damping decrease ratio is 56.54%.

The effect of charge pressure on performance is shown in Fig. 12A–D, respectively. By means of adjusting working param-

eters repeatedly, the lowest temperature obtained under 15 bar is 51 K. When increasing charge pressure, the amplitude of pressure wave will also increase which leads to the increase of cooling capacity and input power simultaneously. In reference to Figs. 10 and 11, the increasing phase shift also contributes to the rise of cooling capacity and input power. But from Eqs. (11) and (12) the COP is just the uniform function of charge pressure. So charging high-pressure gas is an effective way to get high efficiency if the ability of pressure resistance is permitted, which depend on the thickness of shell and material property. But it should be point out that increasing rate of COP slow down when the charge pressure is high (as shown in Fig. 12D). The evaluation of advantage and disadvantage of increasing of charger pressure is naturally very necessary for an optimum design.

5.4. Effect of the natural frequency on the phase shift

In this study, in order to change the natural frequency of displacer, the additional mass was attached to the displacer. 1.95 g

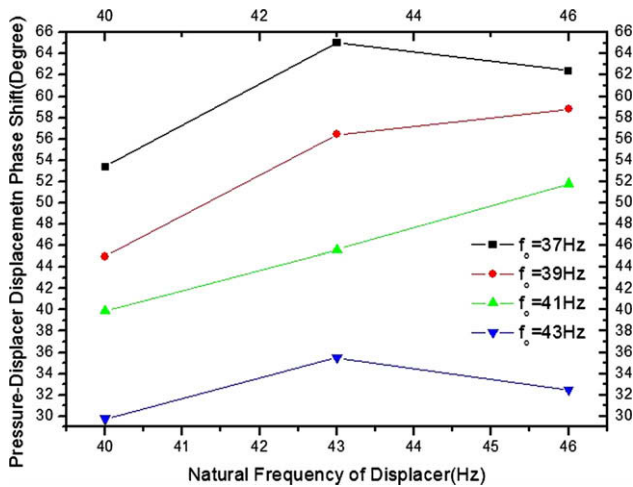


Fig. 14. Phase shift between the displacer displacement and system pressure with the variation of natural frequency of displacer, while $P_c = 15$ bar, $W_{cl} = 300 \mu\text{m}$.

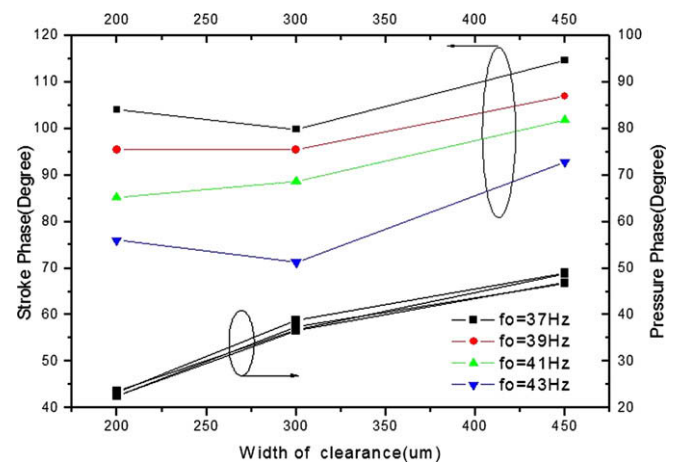


Fig. 16. Displacement phase shift between displacer and piston and phase shift between the system pressure and displacement of piston with the variation of clearance width, while $P_c = 15$ bar, $f_n = 46$ Hz.

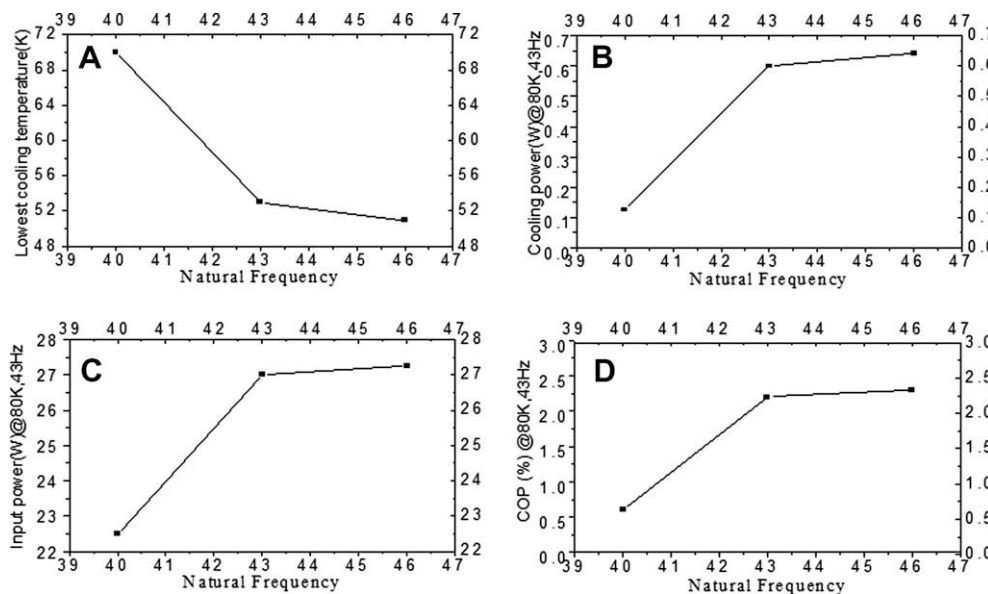


Fig. 15. When $P_c = 15$ bar, $W_{cl} = 300 \mu\text{m}$, with the variation of natural frequency (A) the lowest reached temperature, (B) cooling capacity when the temperature is 80 K, (C) input power when the temperature is 80 K, working frequency is 43 Hz, (D) COP when the temperature is 80 K, working frequency is 43 Hz.

and 7.76 g additional weight parallels to 43 Hz and 40 Hz natural frequency, respectively.

As shown in Fig. 13, in terms of the displacement phase shift, for different three natural frequency of displacer the higher the working frequency is, the lower phase shift will be. If $\omega = \omega_d$ (displacer is turned to the running frequency), then $\phi_{pd} \neq 0$, which do not coincide with the prediction given by Jonge [10]. Actually, the theory developed by Jonge is the special case of Eq. (17) when the gas spring is ignored. In the real test, compressor running full stroke is very dangerous, at this moment the autocontrol system and anti-collision function has been cancelled and substituted by manual operation. So in order to avoid the damage of collision, the full stroke is practically rare. In this case, when the running frequency equal to the natural frequency, positive displacement phase shift still exist and have inverse proportion with the working frequency. For instance, when natural frequency of displacer is 43 Hz, displacement phase shift is 35.5° under the condition of 43 Hz working frequency.

Fig. 13 indicates that pressure phase is not sensitive to the working frequency. And it can also found that when natural frequency of displacer is 43 Hz, the maximum pressure phase shift

(44°) happens under the condition of 41 Hz working frequency. From Eq. (20), when working frequency of compressor equals to that of displacer, the pressure phase reach its peak. According to the vibration theory, the resonant frequency is actually less than natural frequency, which implies natural frequency of piston less than 41 Hz. As a result, it can be drawn from Fig. 13 that natural frequency of compressor should be less than that of displacer, which is the basic principle followed in real design (Fig. 14).

As far as expansion phase is concerned, when natural frequency of displacer is 40, 43 and 46 Hz respectively, 37 Hz working frequency can lead to comparative bigger phase shift to different natural frequencies than 39, 41 and 43 Hz. In this case, the frequency ratio (ω_o/ω_n) is 0.925, 0.86, 0.8 correspondingly and 0.86 can bring up maximum phase shift that approach to the optimum range of frequency ratio (0.75–0.85) provided by Park [14]. Furthermore, the reason why expansion phase shift do not become zero when working frequency equal to resonant frequency is gas stiffness of expansion, especial hot space do not involve into the calculation of natural frequency. Similar to displacement phase shift, this ignorance can cause additional phase shift.

Fig. 15 shows that natural frequency has obvious effect on the performance of cryocooler. For the displacer with natural fre-

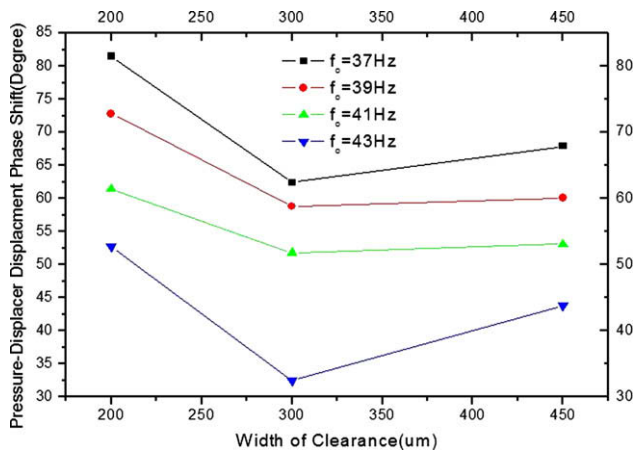


Fig. 17. Phase shift between the displacer displacement and system pressure with the variation of clearance width, while $P_c = 15$ bar, $f_n = 46$ Hz.

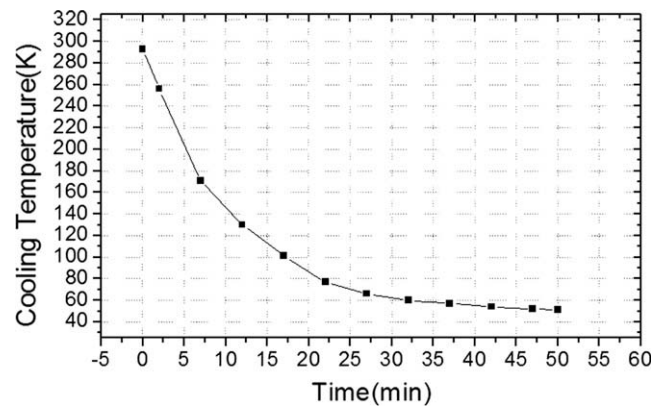


Fig. 19. Cooling curve under the optimum operating conditions.

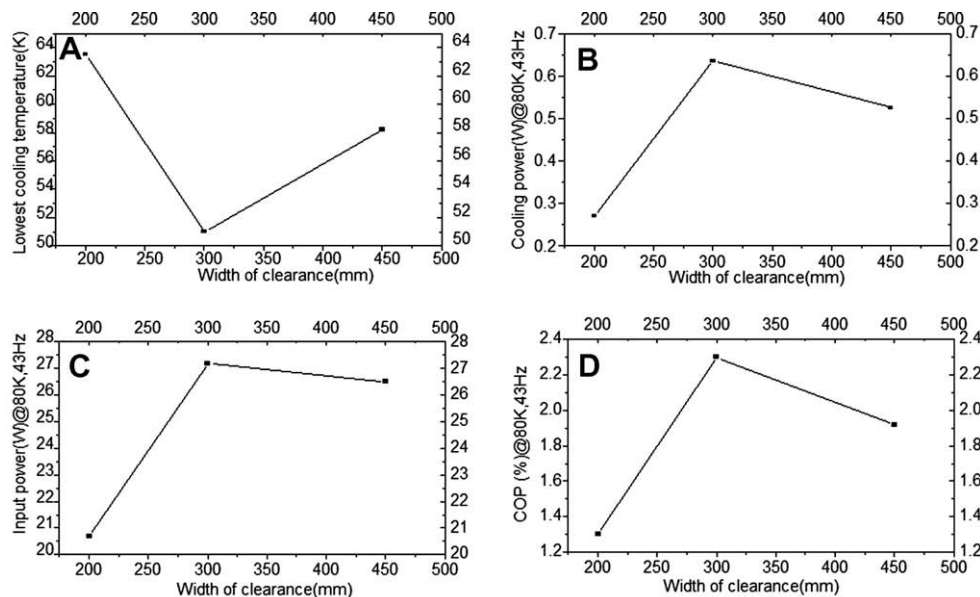


Fig. 18. When $P_c = 15$ bar, $f_n = 46$ Hz, with the variation of clearance width (A) the lowest reached temperature, (B) cooling capacity when the temperature is 80 K, (C) input power when the temperature is 80 K, working frequency is 43 Hz, (D) COP when the temperature is 80 K, working frequency is 43 Hz.

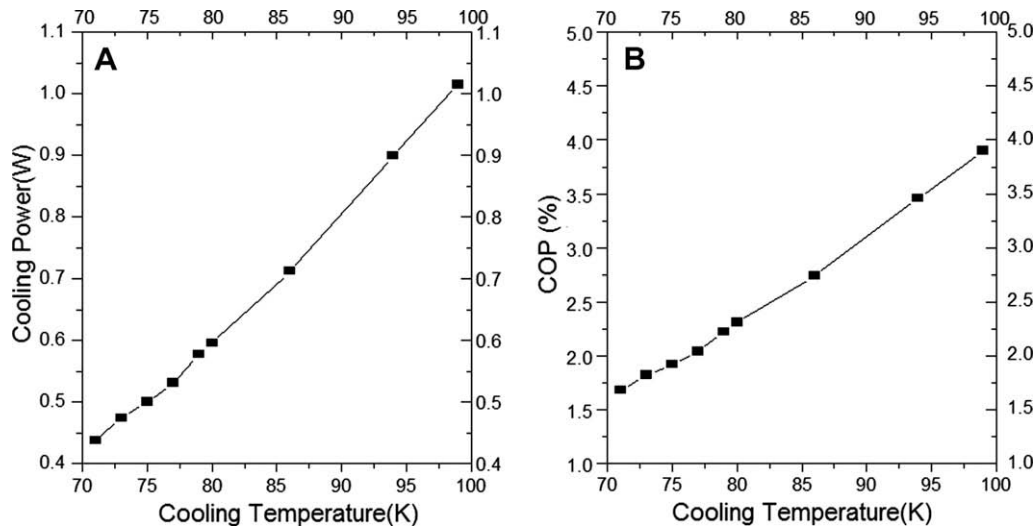


Fig. 20. (A) Cooling temperature vs. cooling power and (B) cooling temperature vs. COP.

quency 46 Hz, when working frequency is 43 Hz, the lowest reached temperature is 51 K and cooling power and input power at 80 K is 0.64 and 27.25 W, respectively. At this point, the displacement phase shift is 71.3° . In refer to Eqs. ((11)–(13)), the increase of amplitude of piston (X_p) and displacer (X_d) lead to decrease of displacement phase shift (θ), similarly the increase of pressure wave causes the decrease of amplitude. So if the maximum cooling power is wanted, the working frequency should be close to the natural frequency of displacer.

5.5. Effect of the damping on the phase shift

Fig. 16 indicates that when working frequency is 41 Hz, displacement phase shift varies from 85.2° to 101.8° with the increase of clearance width from 200 to 450 μm . As predicted by Eq. (17), increasing damping of displacer cause decreasing displacement phase shift. As for pressure phase shift, when damping of displacer increases as a result of decreasing clearance width, from Eq. (18), the amplitude ration will decrease; the stroke of piston and displacer will all increase consequently. In the end, according to Eq. (20), the pressure phase shift will increase as shown in Fig. 16. Experimental data finally verified this conclusion. For instance, when working frequency is 41 Hz and width of clearance vary from 200 to 450 μm , the piston stroke increase from 3.76 to 3.84 mm, correspondently the stroke of displacer increase 1.923 times. As for expansion phase shift, with the increase of clearance, the damping provided decrease; in consequence the phase shift increase. Discrepancy between experiment data and theory attributes to the rigging error (see Figs. 17 and 18).

For 43 Hz working frequency and 46 Hz natural frequency, the lowest temperature obtained is 51 K. Cooling power and input power for 80 K cold tip temperature is 0.635, 27.2 W respectively. From Eqs. (11) and (12), the cooling power and input power have actually close relationship with displacement of piston, displacer and phase shift between them, all of which can be influenced by the damping coefficient of displacer. Maximum COP 2.3 indicates that 300 μm clearance is optimum width in comparison to others.

5.6. Optimum operating condition of Stirling cryocooler

From above experimental analysis, the optimum working conditions for 80 K cold tip temperature is charge pressure 15 bar, natural frequency of displacer 46 Hz, width of clearance 300 μm and working frequency 43 Hz. At this condition 0.632 W cooling power

is obtained at the cost of 27 W input power and COP is 2.31%, lowest obtained temperature is 51 K. Fig. 19 shows cooling curve verse time under optimum operating conditions. It can be seen that from ambient temperature to 77 K, 22 min is needed; 50 min is spent to get the lowest temperature 51 K. The cooling rate of initial stage is faster than that of later stage. Fig. 20A shows the effect of cooling temperature on cooling power under the 26 W input power, the approximate linear relationship exist between them and slope ratio of this line is 0.02 W/K within the range from 71 to 100 K. Similar to cooling power, COP also is almost linear function of cooling temperature at same temperature range. The decline rate of COP against cooling temperature is 0.0794%/K.

6. Conclusions

Phase shift characteristics including displacement, pressure and expansion phase difference have been studied theoretically and experimentally, respectively. Here, the main conclusions have been drawn in the following:

- (1) According to demands of maximum cooling power and COP, the optimum working conditions of this Stirling cryocooler for 80 K cold tip temperature are as follows: charge pressure 15 bar, natural frequency of displacer 46 Hz, width of clearance 300 μm and working frequency 43 Hz. Compared with working frequency, stroke phase and some other parameters, actually phase shift is the most complicated one involving relative movement of two pistons. So in the process of optimum seeking, an indispensable step is to locate the working point in the optimum range of phase shift that can actually act as the index of optimization. As for displacement phase shift, neighborhood interval of 90° is the ideal working domain. Meanwhile, the expansion phase shift should be adjusted approaching to 0° as near as possible. Pressure phase shift, which reflects the damping characteristic of compressor, should also be as small as possible.
- (2) Eqs. (17), (20) and (25) reveal the relationship between three phase shifts and working parameters respectively, predication from which agree with the experimental result qualitatively. Due to the difficulties of measuring pneumatic damping coefficient precisely and coupling characteristic of different parameters highly, quantitative analysis must resort to abstruse mathematic method, which is very hard

to be used in practical adjustment. Conversely, these equations describe the phase shifts function clearly and are able to guide the optimum seeking directly and effectively. Furthermore, the discrepancy between previous theory and current experimental phenomenon has been avoided in Eq. (17) by involving the gas spring stiffness into the mathematic model. The dimensionless damping has been expressed as the function of pressure phase shift, which can be served as the index of performance of compressor.

- (3) By experimental study, many running characteristics have been inferred from the variation of phase shift. The variation of helium property caused by decreasing cold tip temperature increases the damping of both piston and displacer. But it seems that the damping of piston is more sensitive to stroke than property variation. And movement characteristic of piston is controlled in larger degree by the external factors including electric voltage and frequency. In contrast, the motion of displacer depends clearly on pneumatic damping coefficient instead of stroke; the increasing working frequency results in stroke decrease of piston and displacer. In terms of piston, the effect of increasing frequency counteracts decreasing damping caused by the decreasing stroke. In contrast the movement of displacer is influenced by frequency to larger extent. As the charge pressure is increased, resonant frequency of the compressor is accordingly increased on the condition of constant piston stroke, but the resonant frequency of the displacer, which is decided by the mechanical spring, is almost constant. And natural frequency of displacer has little effect on the movement of piston. If damping coefficient of displacer keeps constant, working frequency should match the natural frequency; damping of displacer has negligible impact on the piston movement. There exists the matching problem between the natural frequency and damping coefficient. Higher natural frequency of displacer should be adopted if the damping coefficient is great.

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